



Ingegneria delle Telecomunicazioni

Satellite Communications

3. LEO, MEO, HEO, what else?

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Kepler's Laws

1. The planets move in a plane; the orbits described are ellipses with the Sun at one focus (1602).
2. The vector from the Sun to the planet sweeps equal areas in equal times (the law of areas, 1605).
3. The ratio of the square of the period T of revolution of a planet around the sun to the cube of the semi-major axis a of the ellipse is the same for all planets (1618).

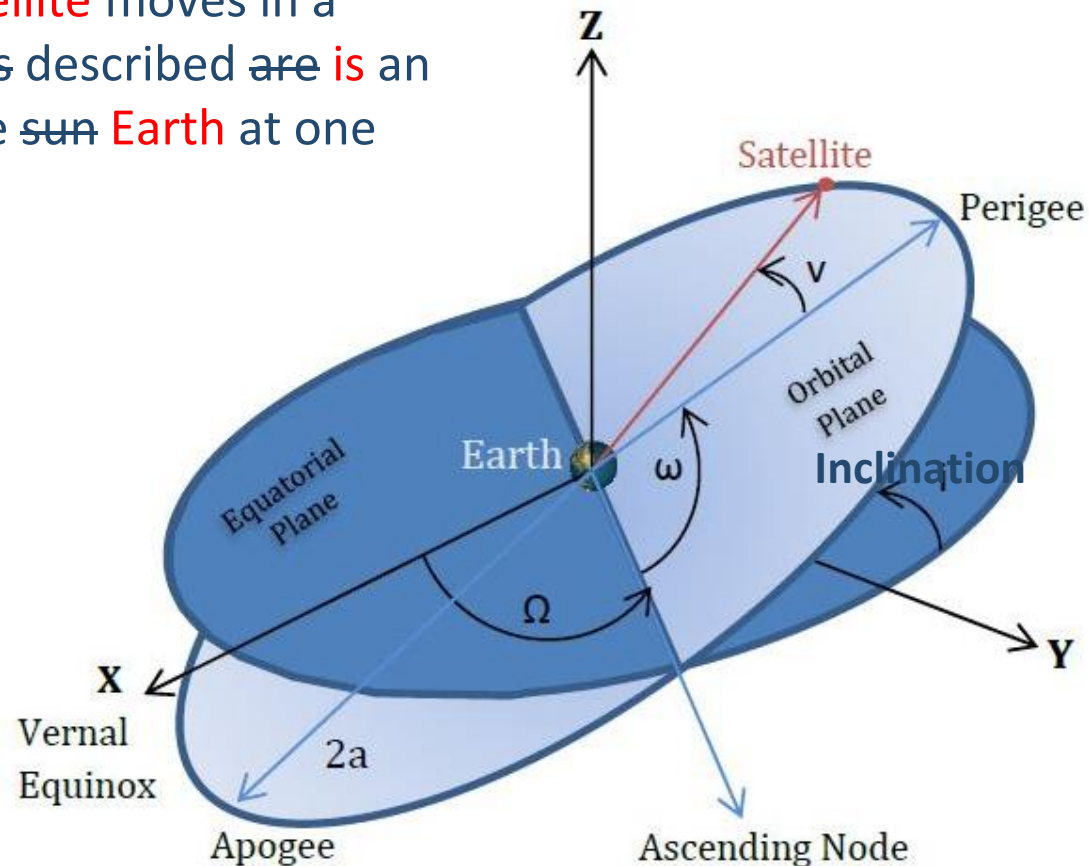


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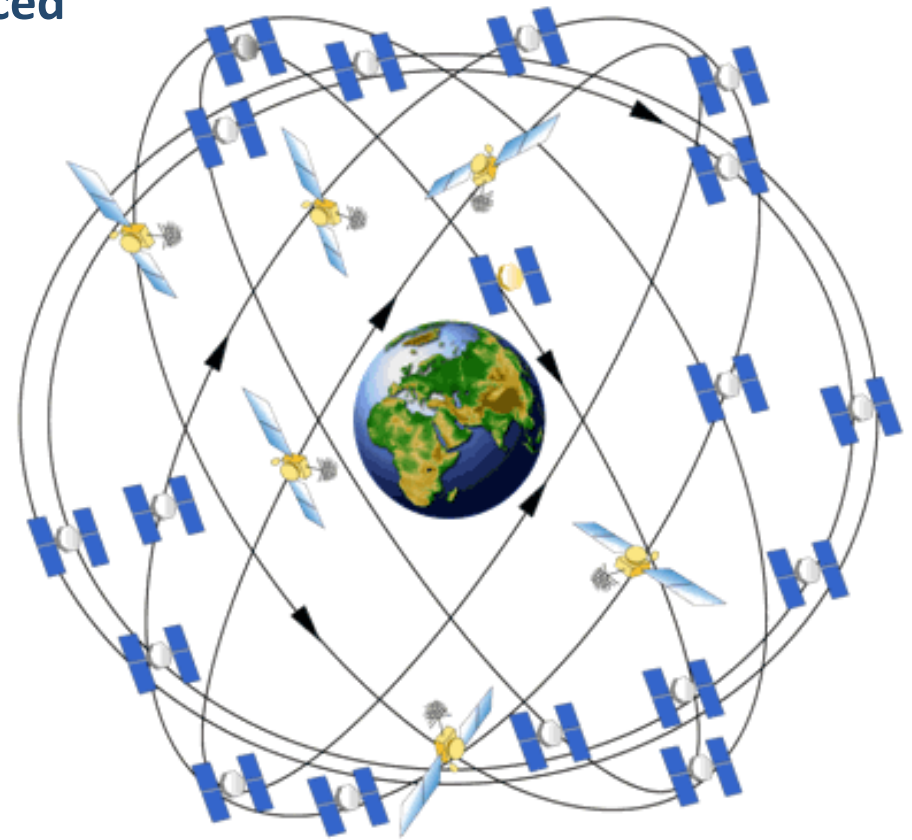
3. LEO, MEO, HEO, what else?

1st Law - Orbital Parameters

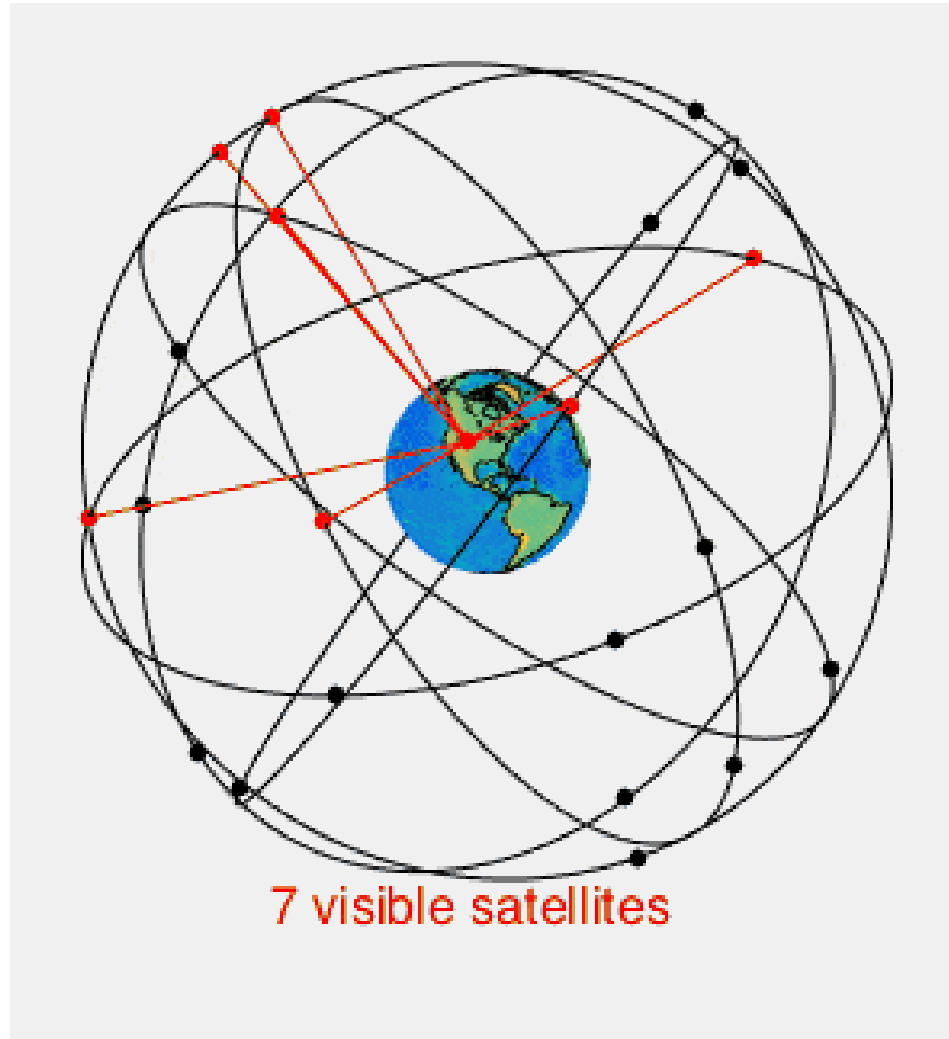
1. The planets **satellite** moves in a plane; the orbits described are **is** an ellipses with the sun **Earth** at one focus



- 24 satellites
- (Almost) Circular Orbits on 6 equi-spaced planes
- Inclination of 55 degrees
- Altitude $h=20,200$ km
- Period (about) 12 hours (11h 58' 2s)
- This 24-slot arrangement ensures users can view at least four satellites from virtually any point of the planet

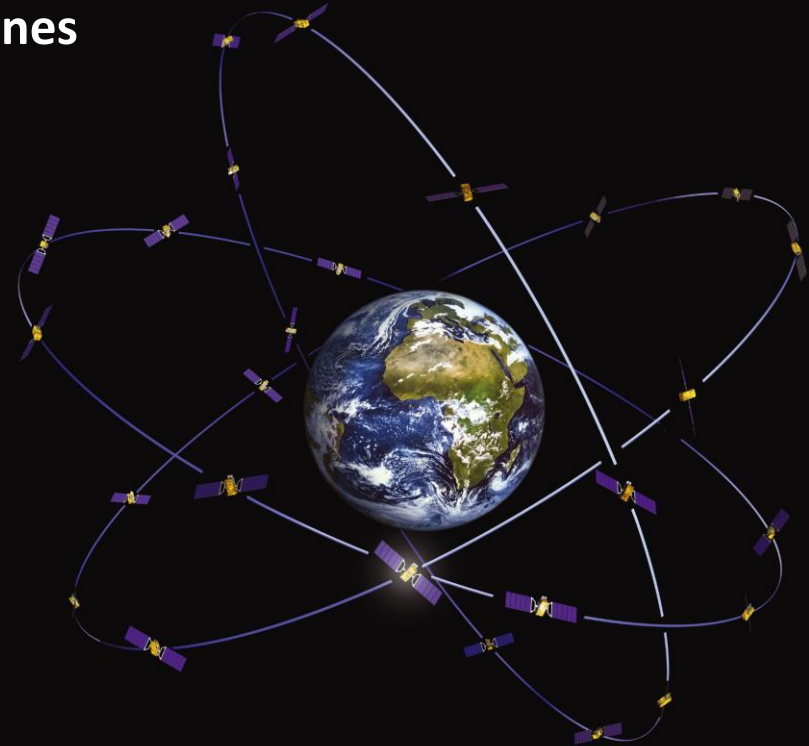


Visibility of GPS Satellites

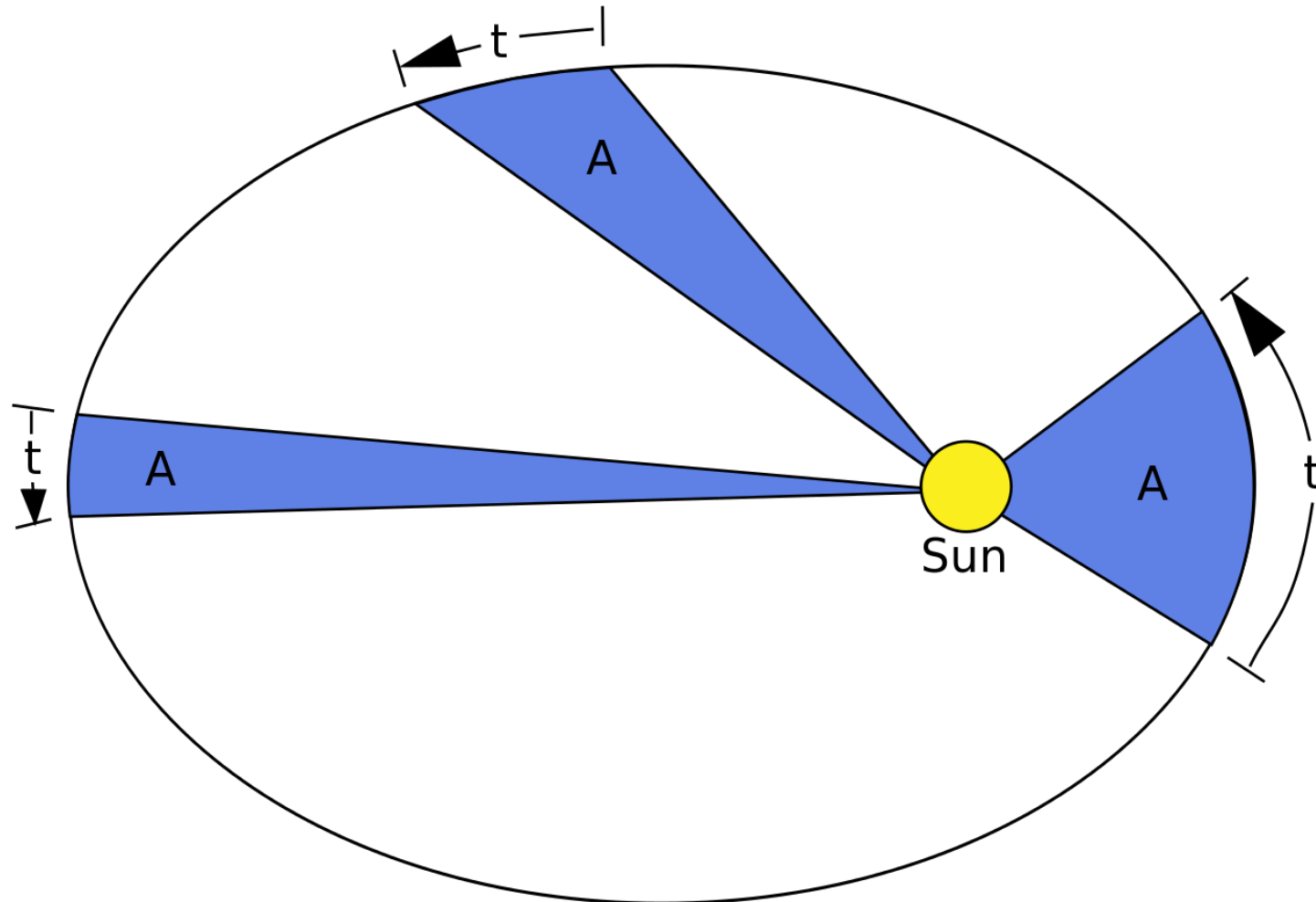


GALILEO Constellation

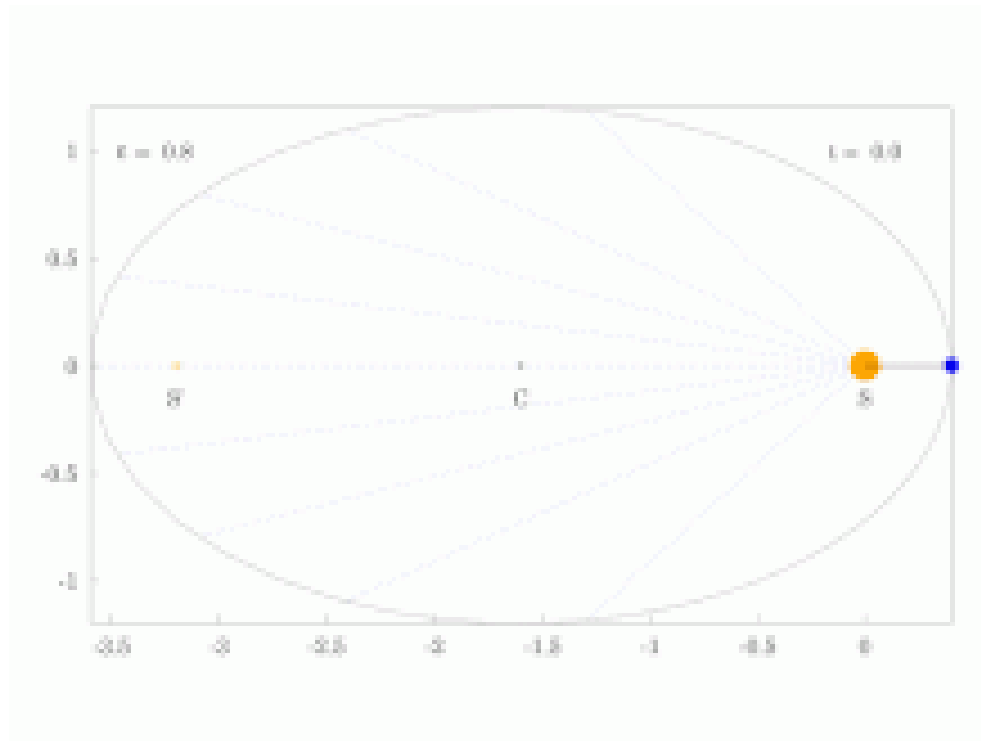
- 24 satellites (+6 spares)
- Circular Orbits on 3 equi-spaced planes
- Inclination of 56 degrees
- Altitude $h=23,222$ km
- Period (about)14 hours
- Better coverage of high-latitudes than GPS



Second Kepler Law



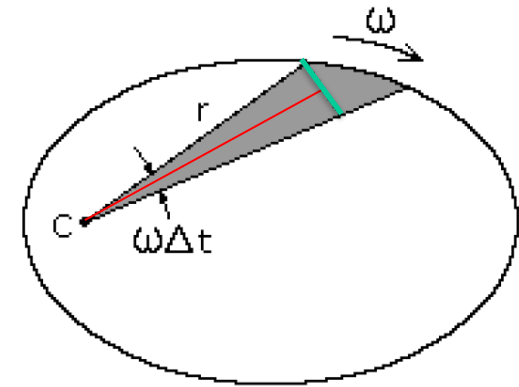
Second Kepler Law



Second Law, revisited

- The vector from the sun to the planet sweeps equal areas in equal times (the law of areas, 1605).

$$\Delta A = r \cdot \left(\frac{r \cdot \omega \Delta t}{2} \right) \Rightarrow \frac{dA}{dt} = \frac{\omega r^2}{2} = \text{constant}$$

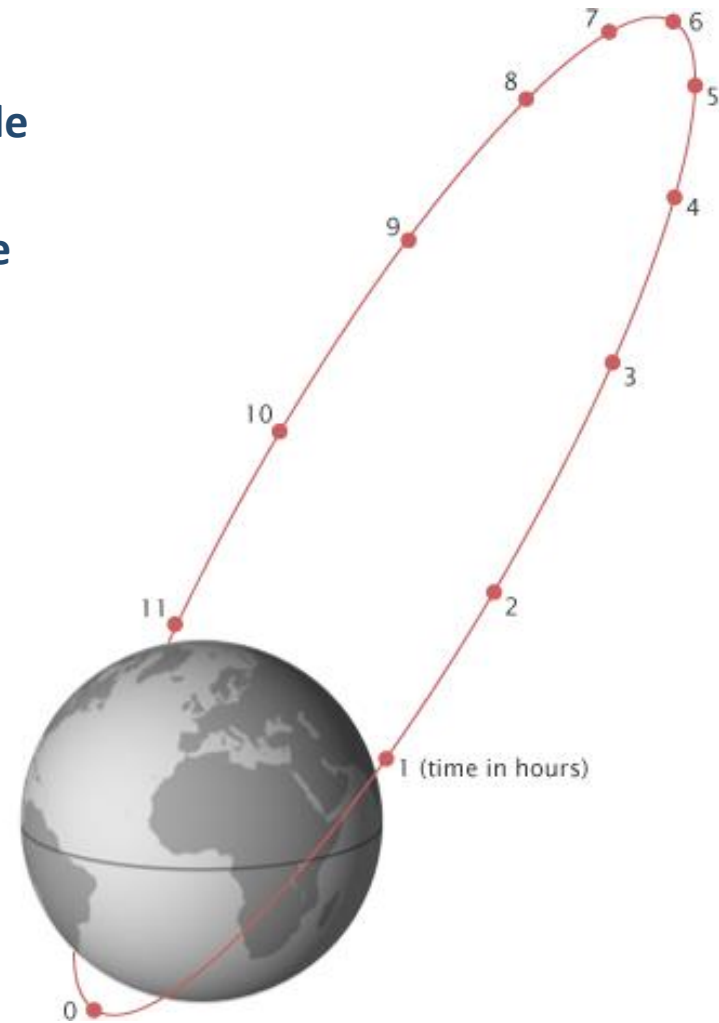


- So another formulation is: @any point

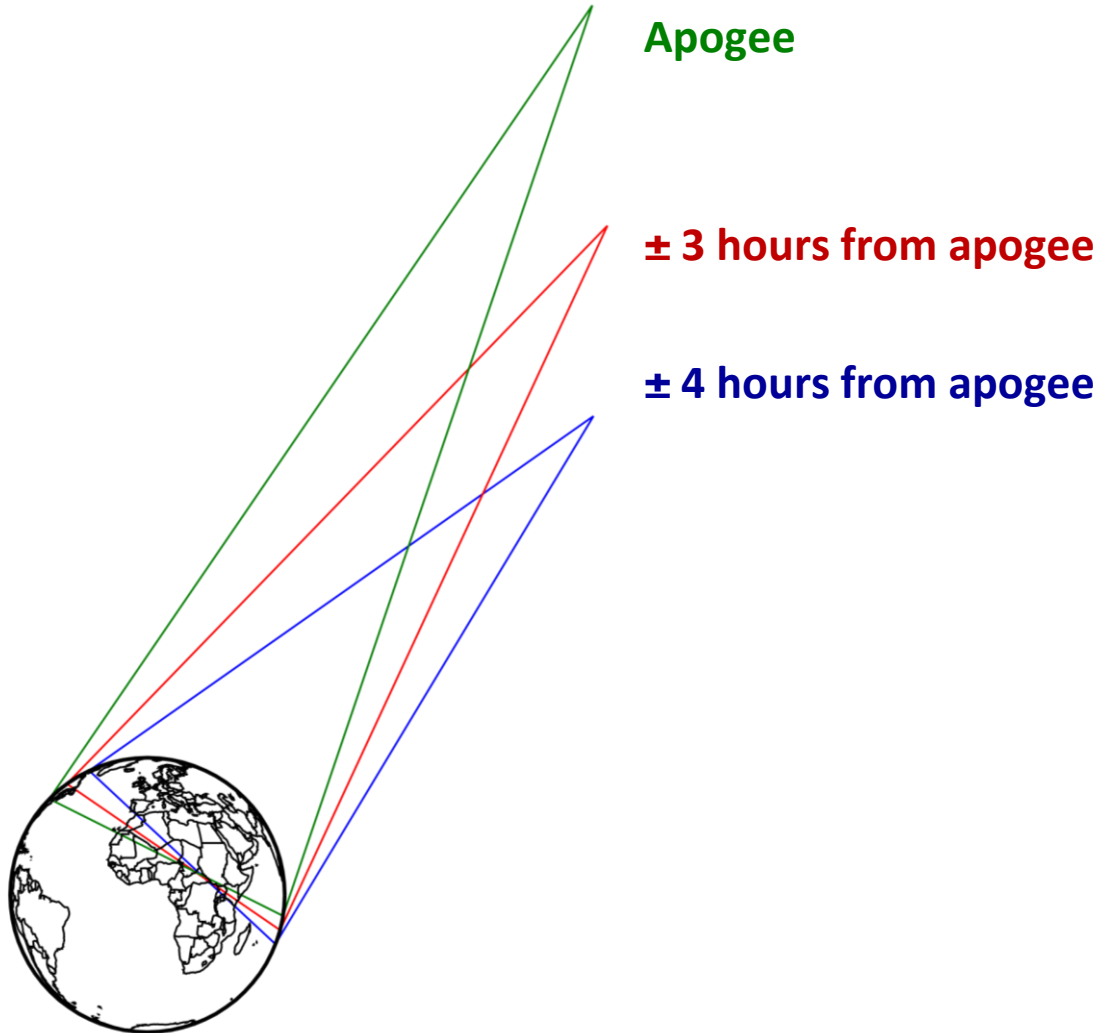
$$\omega r^2 = \text{constant}$$

Molniya Orbit

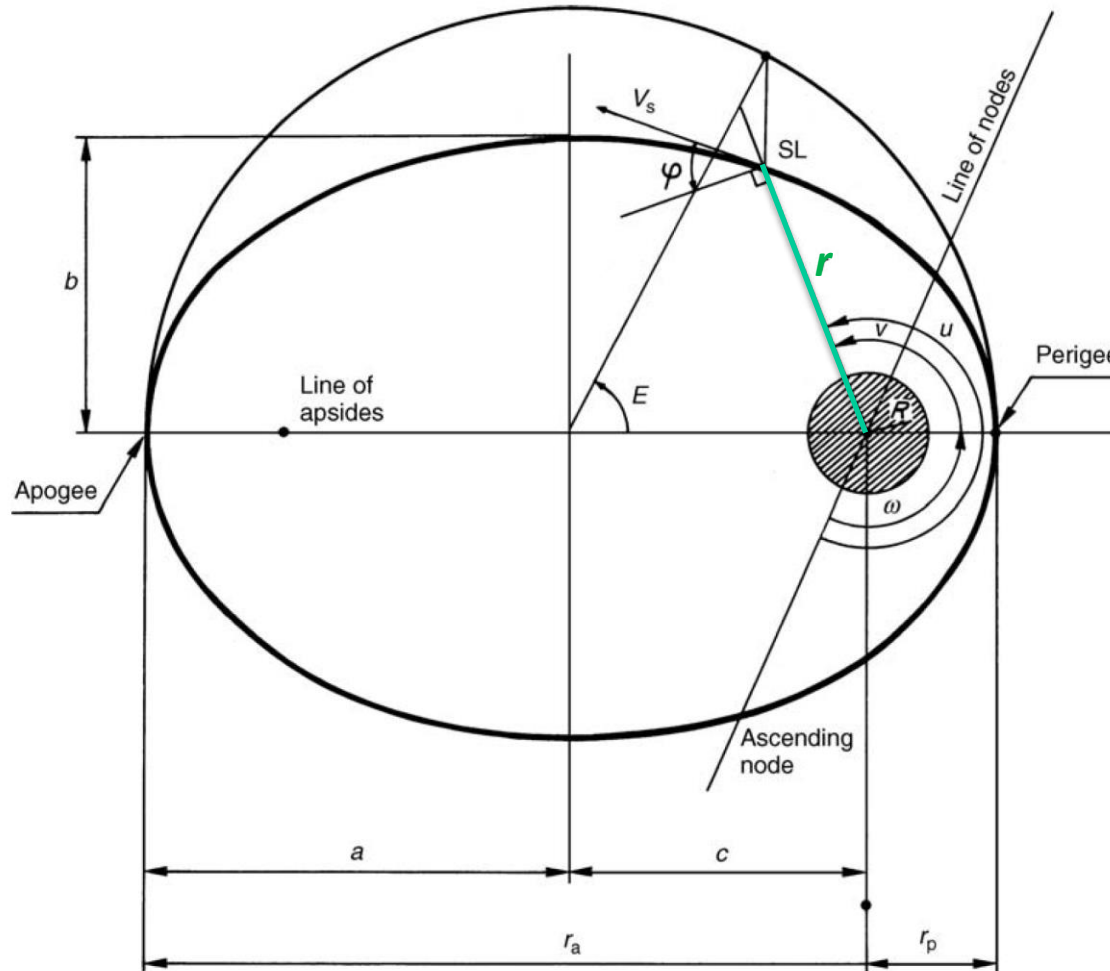
- Russian Satellites to cover very high-latitude areas
- The satellite stays at apogee for a long time (2nd law...)



Coverage of Molniya



Parameters of the Orbit



a, b semi-major and semi-minor ellipse axes

$$c = (a^2 - b^2)^{1/2}$$

r_a, r_p altitude of apogee/perigee

$$r_p = a - c, \quad r_a = a + c$$

$e = c/a$ eccentricity

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \frac{r_a - r_p}{r_a + r_p}$$

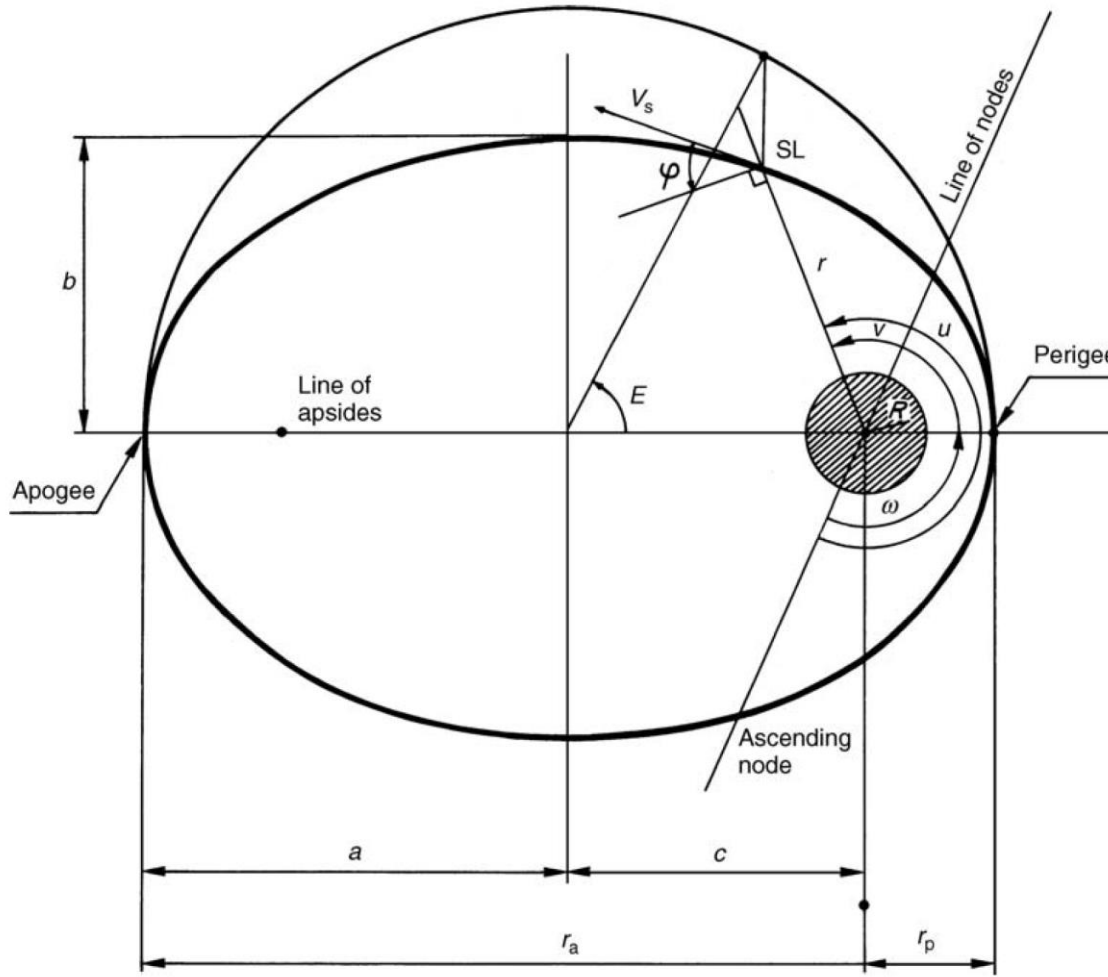
Orbits of Planets

Table 1. Orbital Data for the Planets

Planet	Semimajor Axis (AU)*	Period (y)	Eccentricity
Mercury	0.39	0.24	0.21
Venus	0.72	0.6	0.01
Earth	1	1.00	0.02
Mars	1.52	1.88	0.09
(Ceres)	2.77	4.6	0.08
Jupiter	5.20	11.86	0.05
Saturn	9.54	29.46	0.06
Uranus	19.19	84.01	0.05
Neptune	30.06	164.82	0.01

* 1 au = 149,597,870.707 km

Parameters of the Orbit



For any point on the elliptical orbit

$$r = a \frac{1 - e^2}{1 + e \cos(v)}$$

And the speed of the satellite is

$$v = \sqrt{GM \left(\frac{2}{r} - \frac{1}{a} \right)}$$

$GM = \mu$ standard gravitational parameter $3.986 \cdot 10^{14} \text{ m}^3/\text{s}^2$

Computation of the Satellite Speed 1/2

- Conservation of energy (kinetic+gravitational potential)

$$\frac{mv^2}{2} + \left(-\frac{GMm}{r} \right) = \text{constant}$$

- @ perigee/apogee, + conservation of angular momentum

$$\frac{v_a^2}{2} + \left(-\frac{GM}{r_a} \right) = \frac{v_p^2}{2} + \left(-\frac{GM}{r_p} \right) \quad mv_a r_a = mv_p r_p \quad \Rightarrow \quad v_p = v_a \cdot r_a / r_p$$

- So

$$\frac{v_a^2}{2} \left(1 - \frac{r_a^2}{r_p^2} \right) = GM \left(\frac{1}{r_a} - \frac{1}{r_p} \right) \quad \Rightarrow \quad \frac{v_a^2}{2} = GM \frac{r_p}{r_a} \frac{1}{(r_p + r_a)}$$

Computation of the Satellite Velocity 2/2

- **But**

$$r_p + r_a = 2a \Rightarrow \frac{v_a^2}{2} = GM \frac{r_p}{r_a} \frac{1}{2a} = GM \frac{2a - r_a}{2a \cdot r_a} = GM \left(\frac{1}{r_a} - \frac{1}{2a} \right)$$

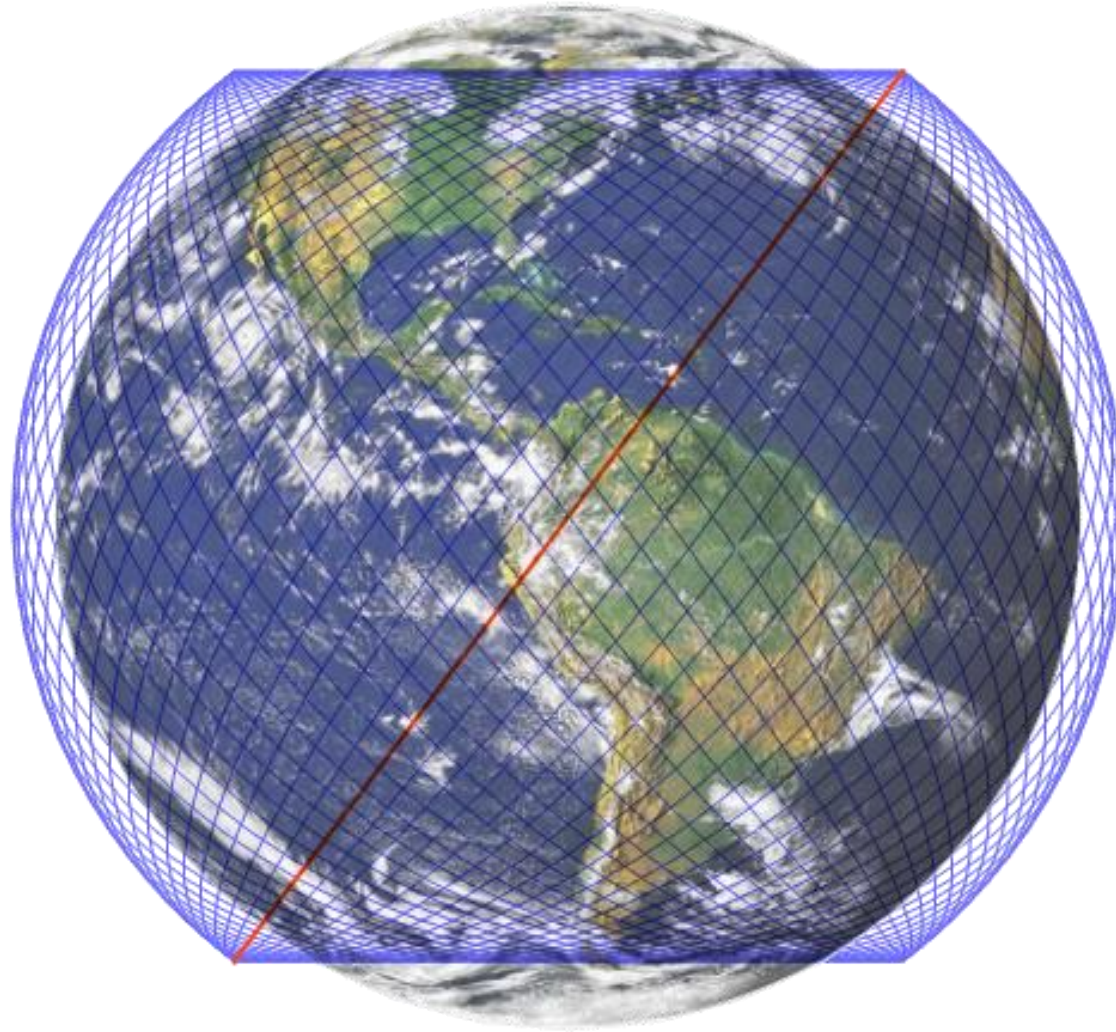
- so we can generalize and get back to the start: for any point

$$\frac{v^2}{2} = GM \left(\frac{1}{r} - \frac{1}{2a} \right) \Rightarrow v = \sqrt{GM \left(\frac{2}{r} - \frac{1}{a} \right)}$$

- **For circular orbits ($r=a$)**

Altitude (km)	Radius (km)	Period (s)	Velocity (m s ⁻¹)
200	6578	5309	7784
290	6668	5419	7732
800	7178	6052	7450
20000	26378	42636	3887
35786	42164	86164	3075

Megaconstellations: StarLink

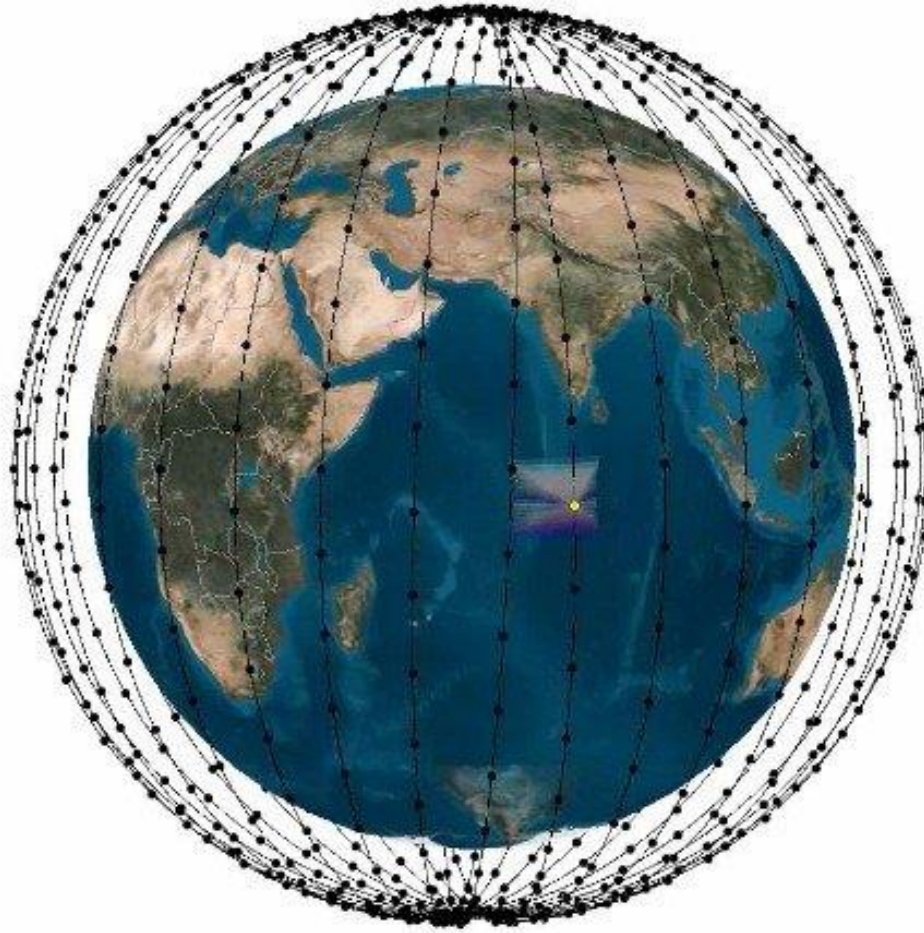


About 3,000 satellites as of Sept. 2022, out of a planned total of FCC-approved 12,000

Nominal altitude: 550 km

Inclination: 53 degrees

Megaconstellations: OneWeb



**Planned total of 684
small satellites**

**Nominal altitude:
1200 km**

**Inclination: 86.4
degrees (quasi-polar)**

As for the Third Law...

- The ratio of the square of the period T of revolution of a planet around the sun to the cube of the semi-major axis a of the ellipse is the same for all planets (1618).

$$T^2 = \alpha \cdot a^3$$

- For the GEO we found

$$G \frac{mM}{r^2} = m\omega^2 r \quad \Rightarrow \quad \frac{GM}{a^3} = \frac{4\pi^2}{T^2}$$

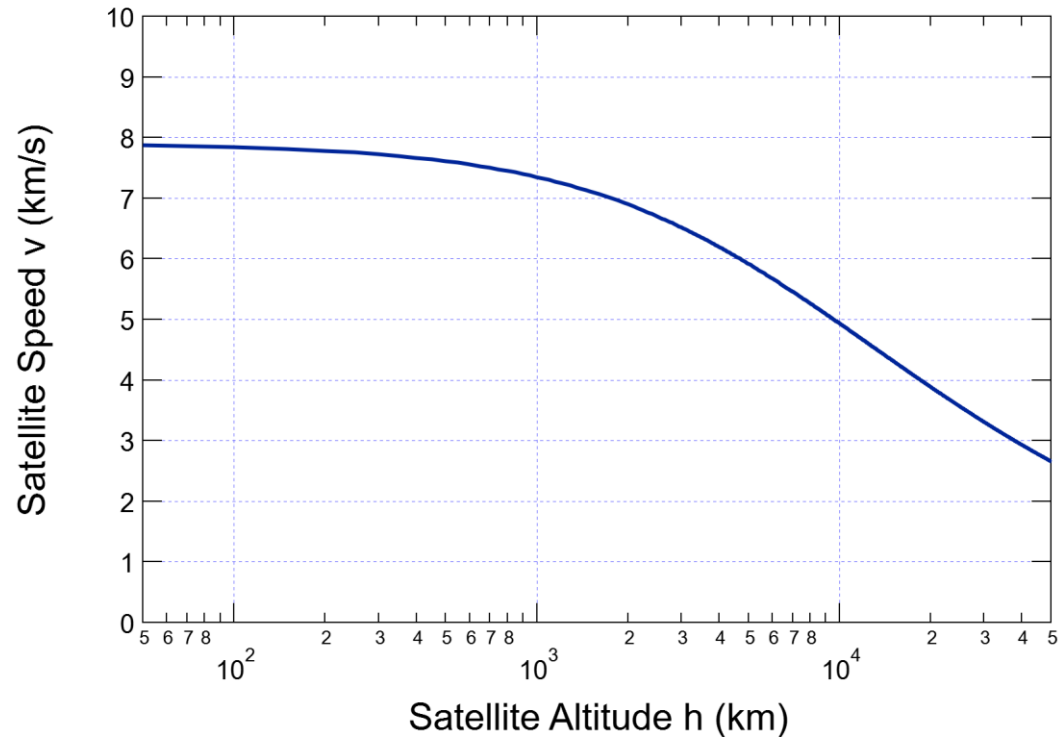
- Therefore, we can find α from the GEO special case:

$$\alpha = \frac{4\pi^2}{GM} \quad \Rightarrow \quad T = 2\pi \sqrt{\frac{a^3}{GM}}$$

As for the Third Law...

$$T = 2\pi \sqrt{\frac{(R+h)^3}{GM}}$$

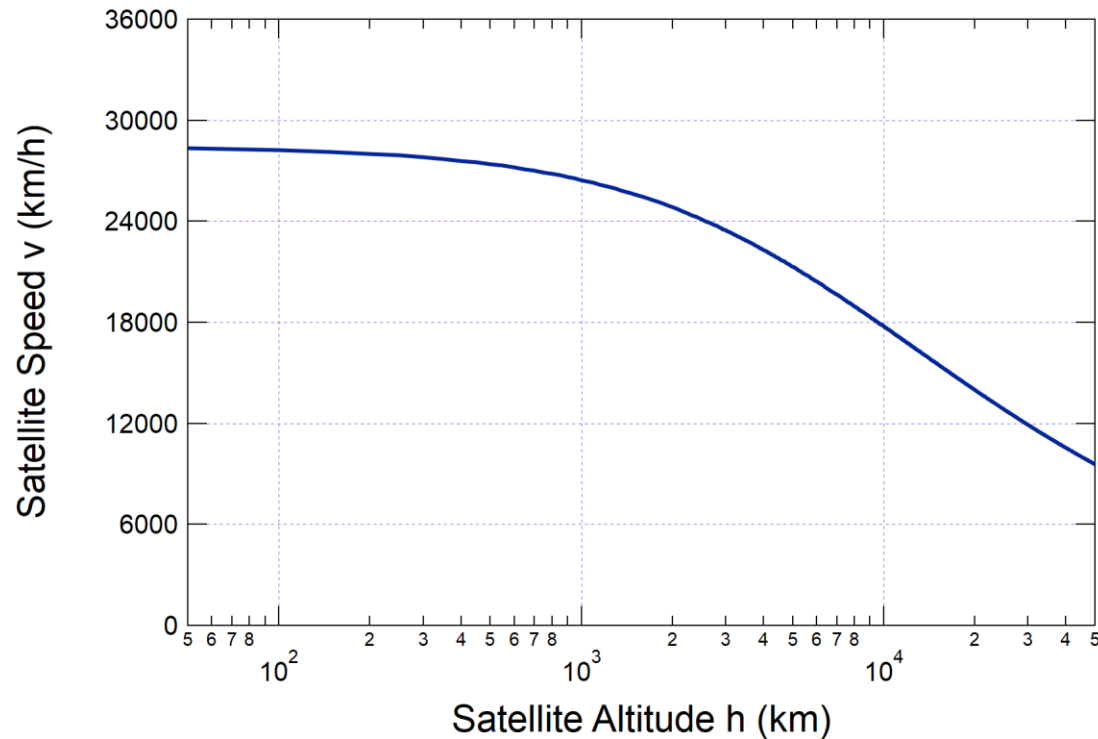
$$v = \sqrt{\frac{GM}{R+h}}$$



As for the Third Law...

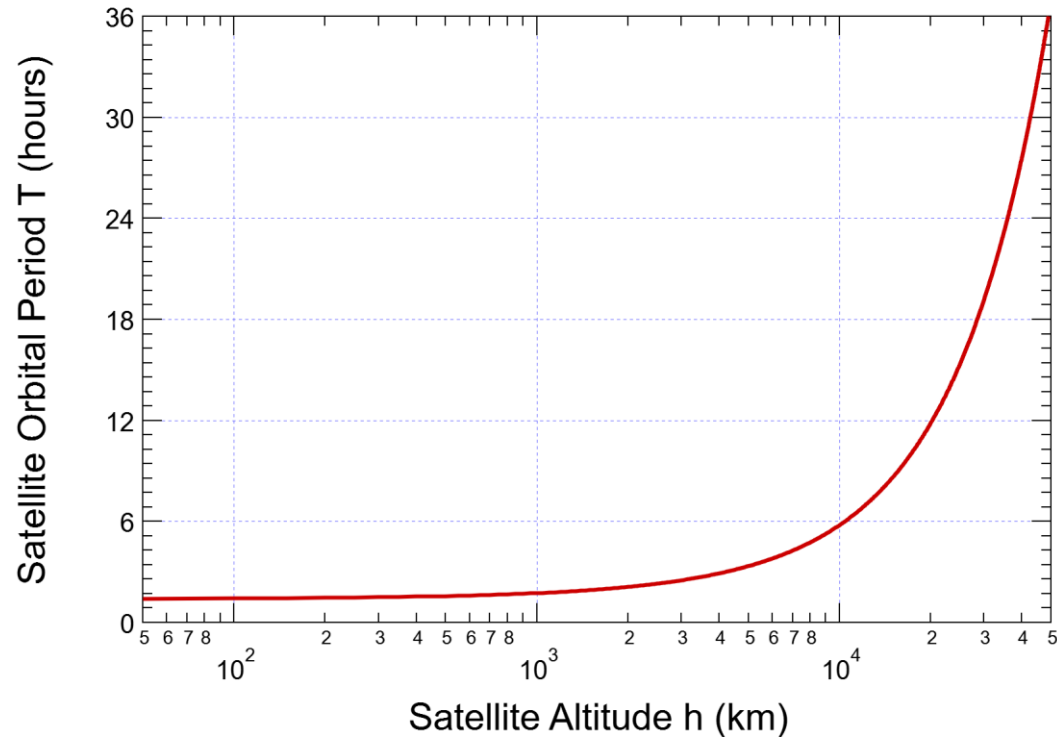
$$T = 2\pi \sqrt{\frac{(R+h)^3}{GM}}$$

$$v = \sqrt{\frac{GM}{R+h}}$$



As for the Third Law...

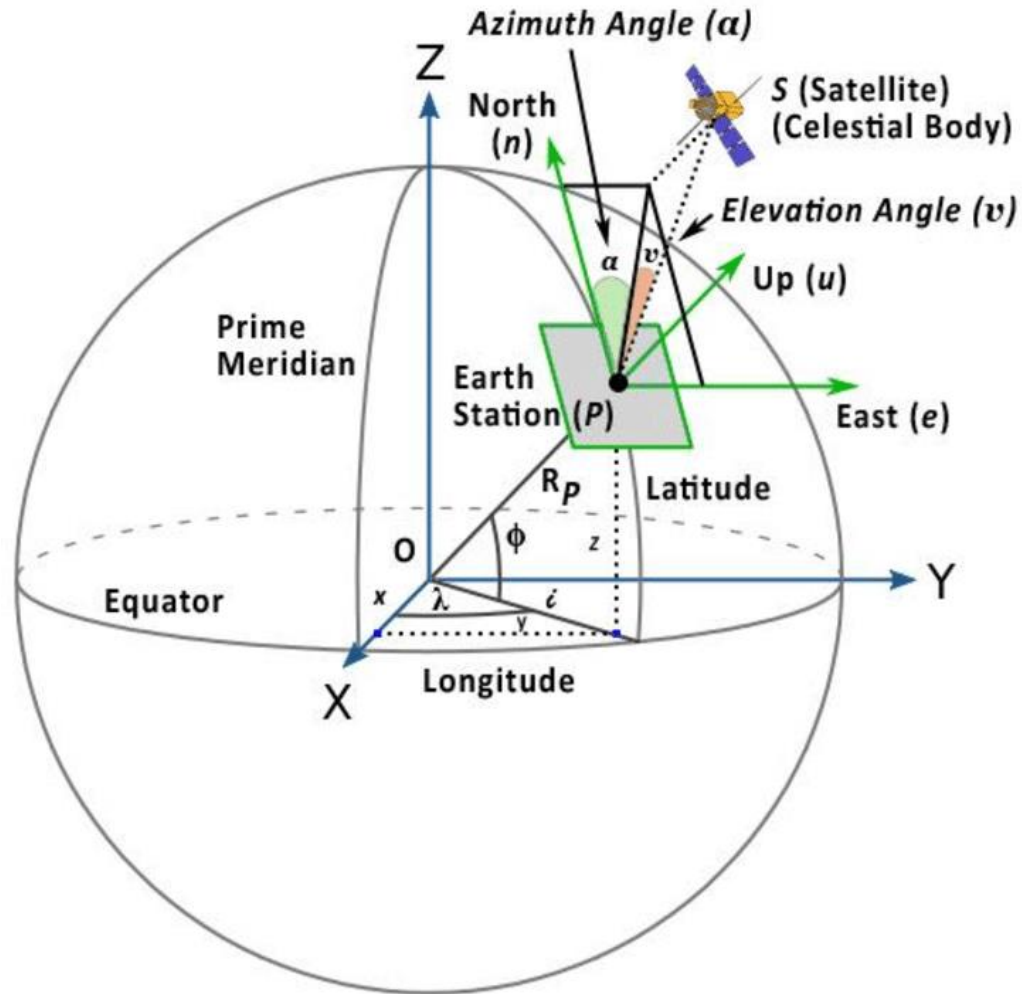
$$T = 2\pi \sqrt{\frac{(R+h)^3}{GM}} \Rightarrow \omega = \frac{2\pi}{T} = \sqrt{\frac{GM}{(R+h)^3}} \Rightarrow v = \omega(R+h) = \sqrt{\frac{GM}{R+h}}$$



- **Coordinates of the Earth Station (terminal)**
 - Left-handed Cartesian coordinate system x,y,z with O at the center of the Earth, the x - y plane coincident with the equatorial plane, and the x -axes lying in the polar plane containing Greenwich
 - Spherical coordinates system r,λ,ϕ with O at the center of the Earth, the *longitude* angle measured wrt the polar plane containing Greenwich, the *latitude* angle λ measured wrt the equatorial plane. Often, the *radius* r is decomposed as $r=R+h$ with h the *altitude* and R the Earth radius
 - Same for the satellite, when needed

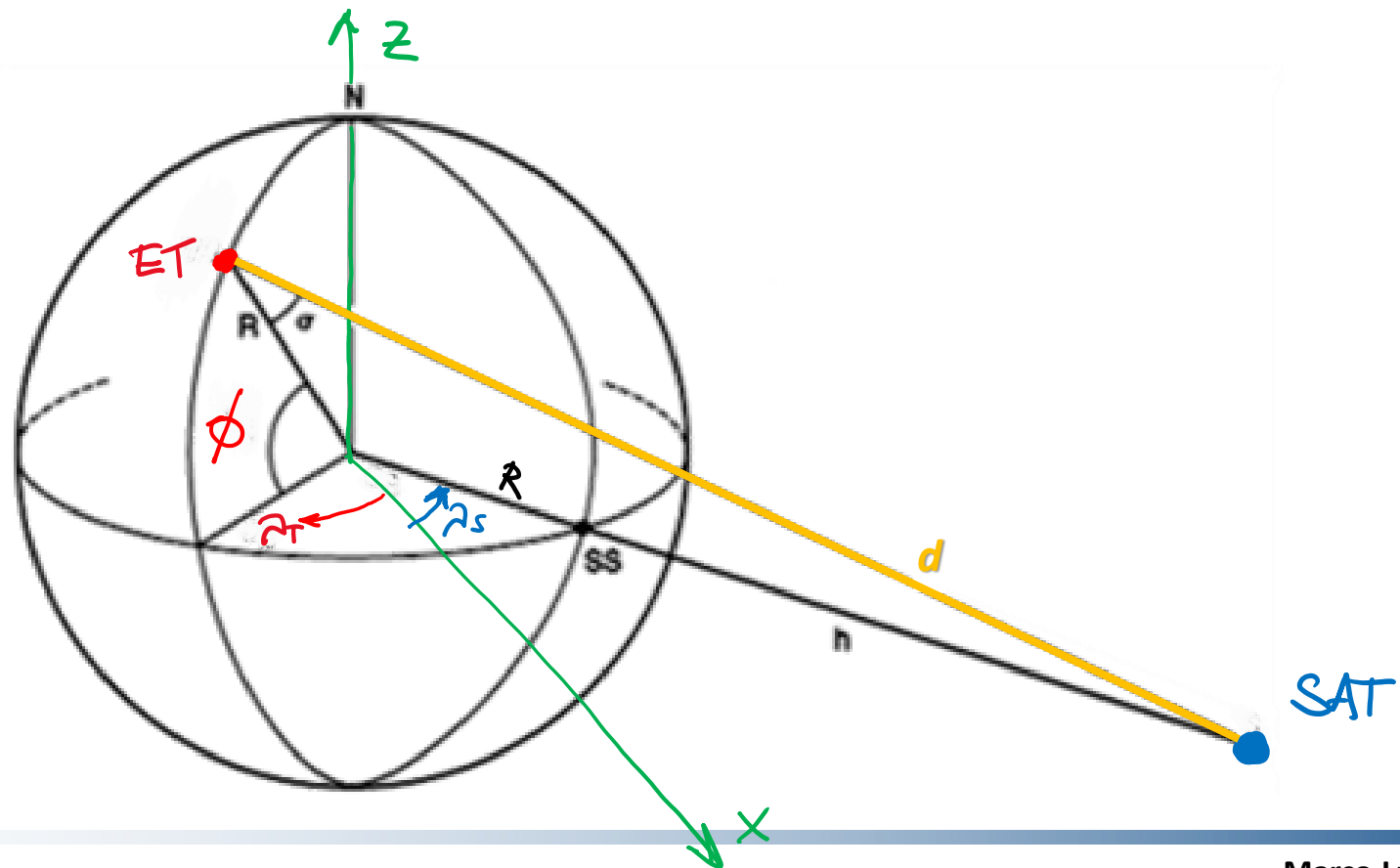
- **Coordinates of the Satellite wrt the Earth Station (ES)**
 - Define the tangential plane to the Earth surface containing the ES, two “local” x - y axes lying on such plane called North and East, the North axes pointing the North pole, and a “local” z axis called Up to make a left-handed system.
 - The “local longitude” is called *azimuth* α and the “local latitude” is called *elevation* ν (sometimes θ). The “local radius” is of course the *altitude* h as above

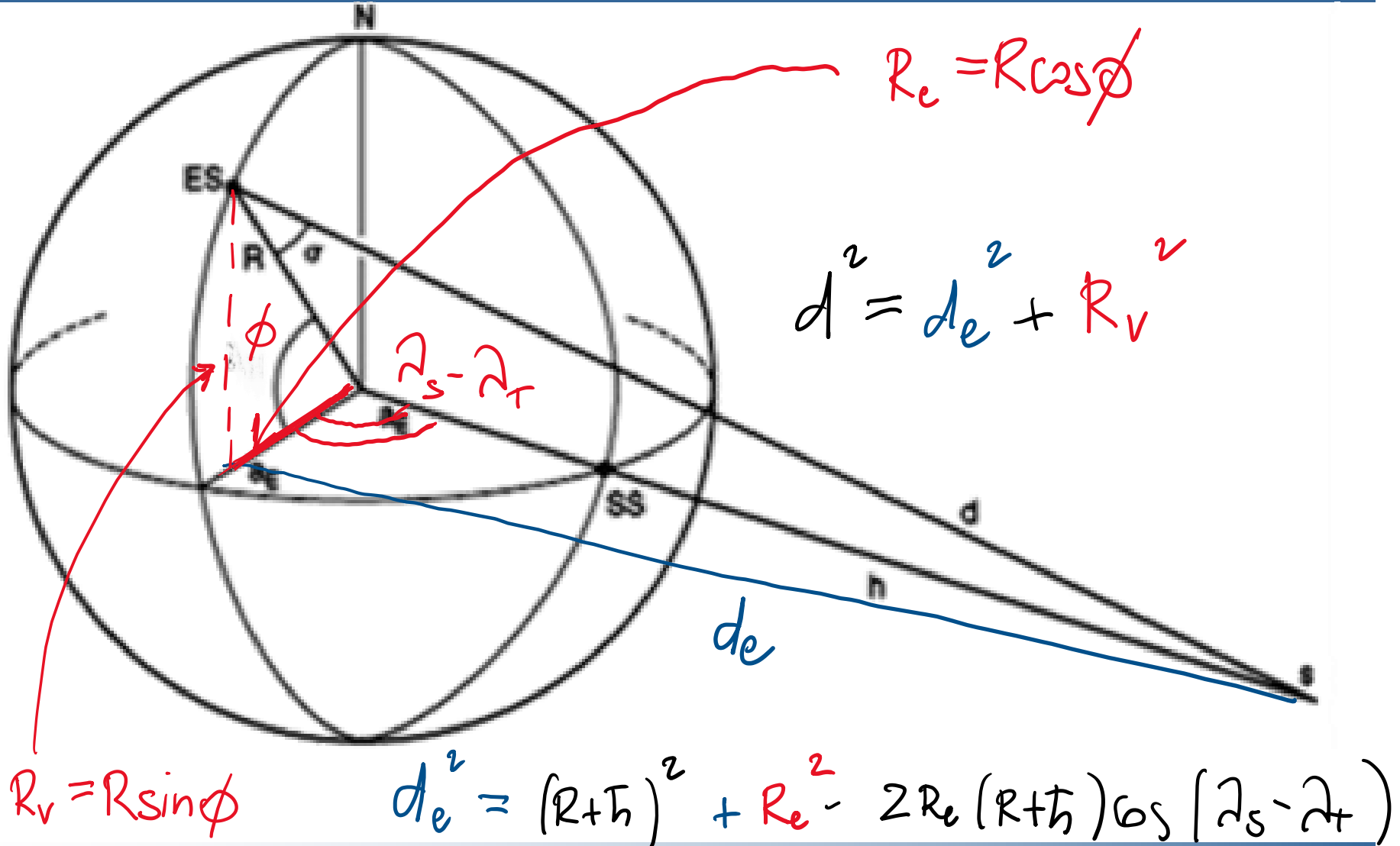
Satellite Angles



The Mother-of-All SatCom Orbits

- Satellite SAT latitude 0, longitude λ_s
- Earth Terminal ET latitude ϕ , longitude λ_T
- We need to compute the ET-SAT distance d



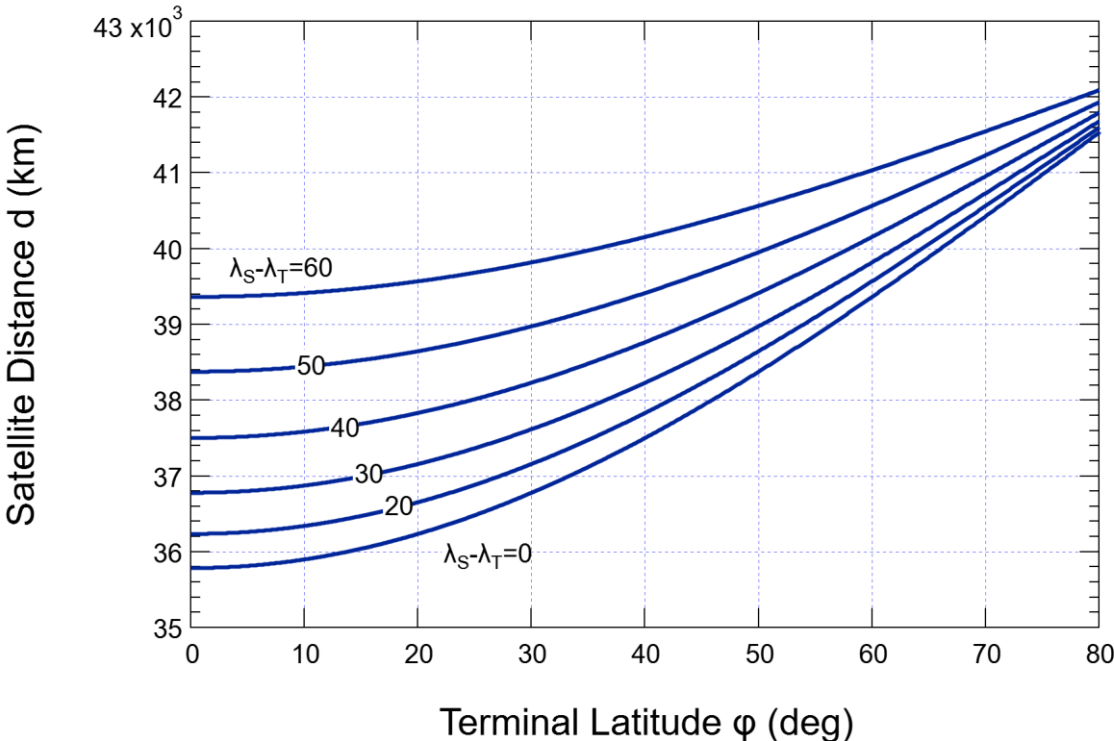
Computation of d 

Computation of d

$$d^2 = (R + h)^2 + R^2 \cos^2(\phi) - 2(R + h)R \cos(\phi) \cos(\lambda_S - \lambda_T) + R^2 \sin^2(\phi)$$

- and finally

$$d = \sqrt{(R + h)^2 + R^2 - 2(R + h)R \cos(\phi) \cos(\lambda_S - \lambda_T)}$$



To aim your dish: Elevation and Azimuth

- **Input:** your latitude λ_T and longitude ϕ , and the SAT longitude λ_S
- **Output:** the elevation ν and azimuth α to mount your parabolic antenna

IT IS COMPLICATED

Elevation:

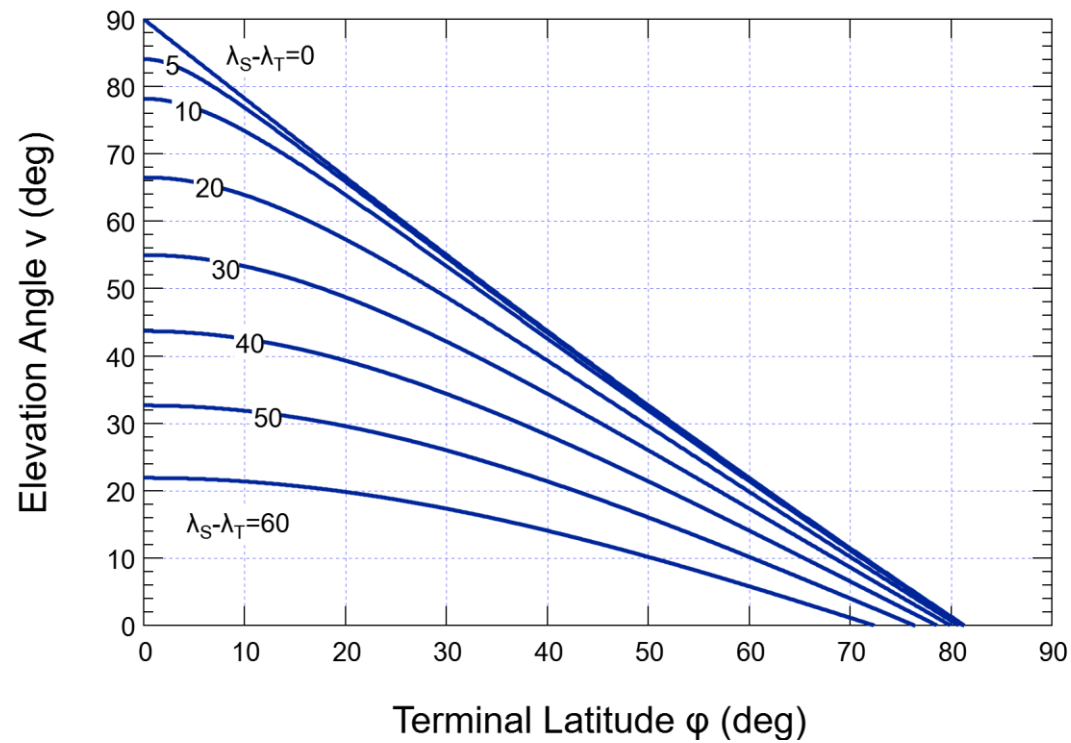
$$\nu = \arccos\left(\frac{R+h}{d}\right) \sqrt{1 - \cos(\phi)^2 \cos^2(\lambda_S - \lambda_T)}$$

Pseudo-Azimuth:

$$\alpha_0 = \arcsin\left(\frac{|\sin(\lambda_S - \lambda_T)|}{\sqrt{1 - \cos(\phi)^2 \cos^2(\lambda_S - \lambda_T)}}\right)$$

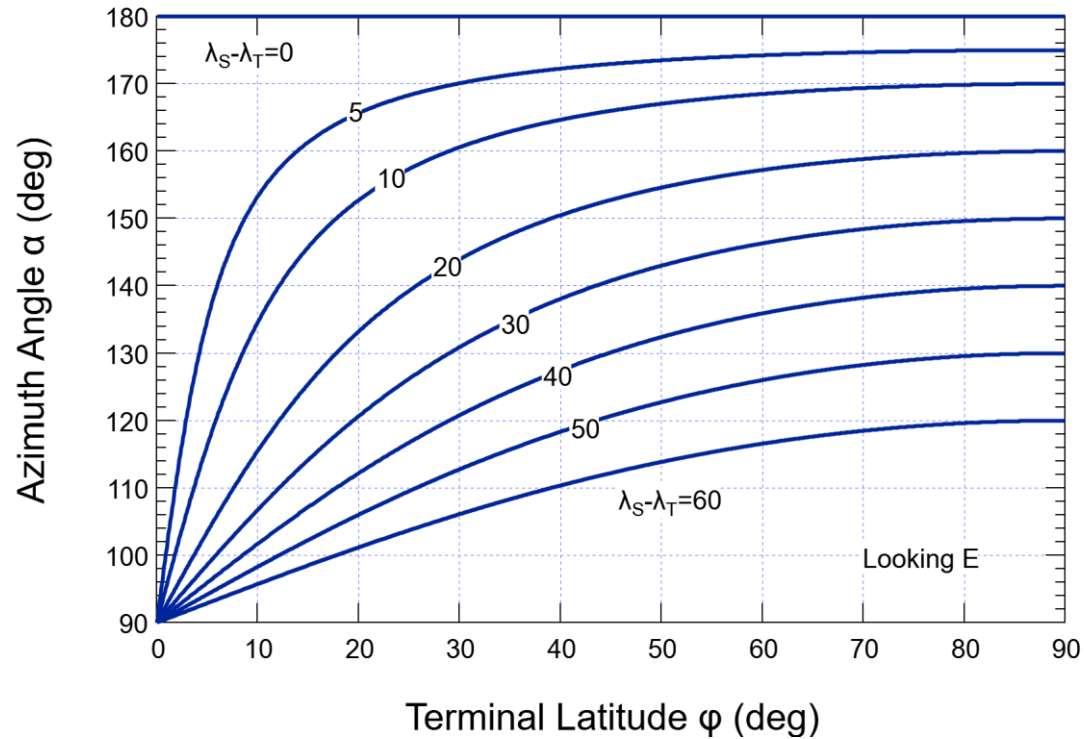
Elevation Chart

$$\nu = \arccos\left(\frac{R+h}{d}\right) \sqrt{1 - \cos(\phi)^2 \cos^2(\lambda_S - \lambda_T)}$$



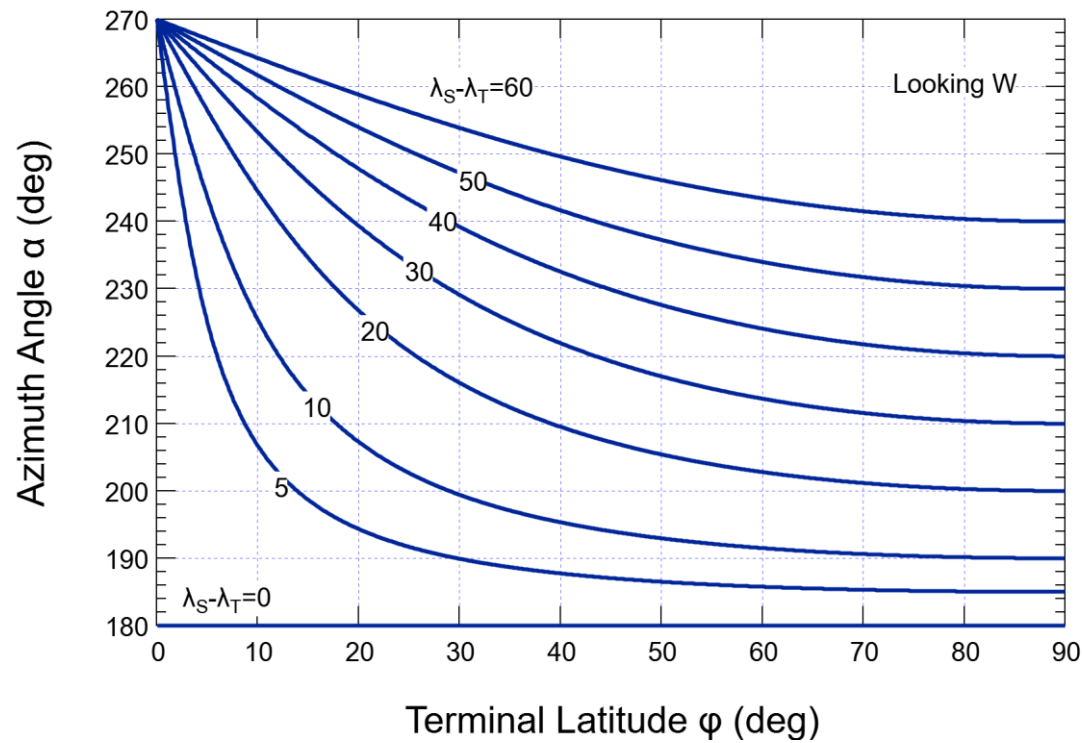
Azimuth Chart (Northern Hemisphere, looking EAST)

$$\alpha = 180 - \alpha_0 \quad (\text{deg})$$



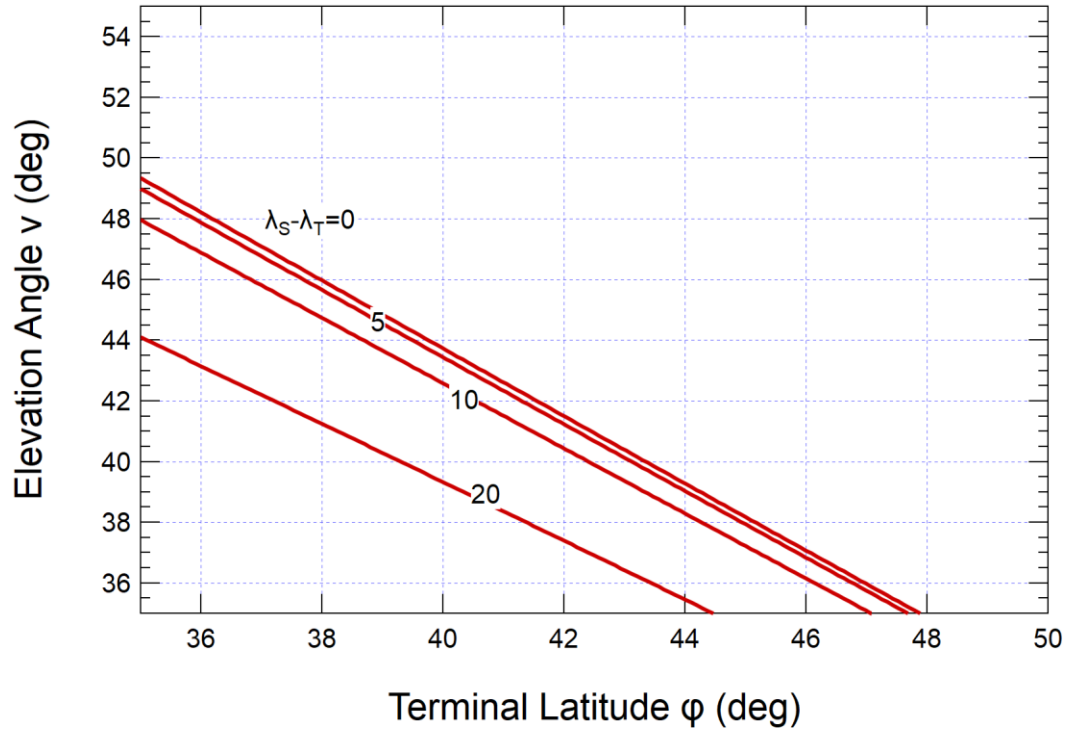
Azimuth Chart (Northern Hemisphere, looking WEST)

$$\alpha = 180 + \alpha_0 \quad (\text{deg})$$



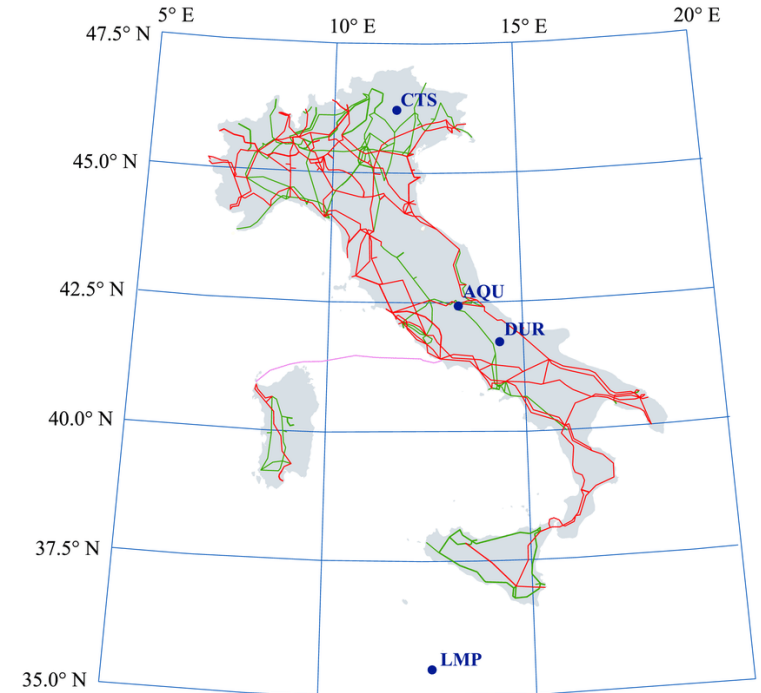
- **Come ricevere SKY a Pisa, Via Caruso (43.72N – 10.38E)**
 - In Italia è possibile ricevere gratuitamente i canali satellitari della piattaforma Tivùsat che replica l'offerta del digitale terrestre (Rai Uno, Rai Due, Rai Tre, Canale 5, Italia 1, Rete 4, La 7), più altri numerosi canali anche in HD e 4K.
 - Tivùsat trasmette dai satelliti HotBird della flotta Eutelsat (13° Est).
 - Anche i canali di Sky vengono diffusi attraverso HotBird 13° Est, e per questo motivo, sul territorio nazionale, la stragrande maggioranza delle parabole sono orientate a sud verso questo satellite.
 - In Italia è anche possibile ricevere i canali trasmessi dal satellite Astra 19,2° Est, che presenta una ricca offerta di televisioni europee e mondiali, con oltre 350 canali gratuiti (tra cui Eurosport).

Elevation Chart for Italy

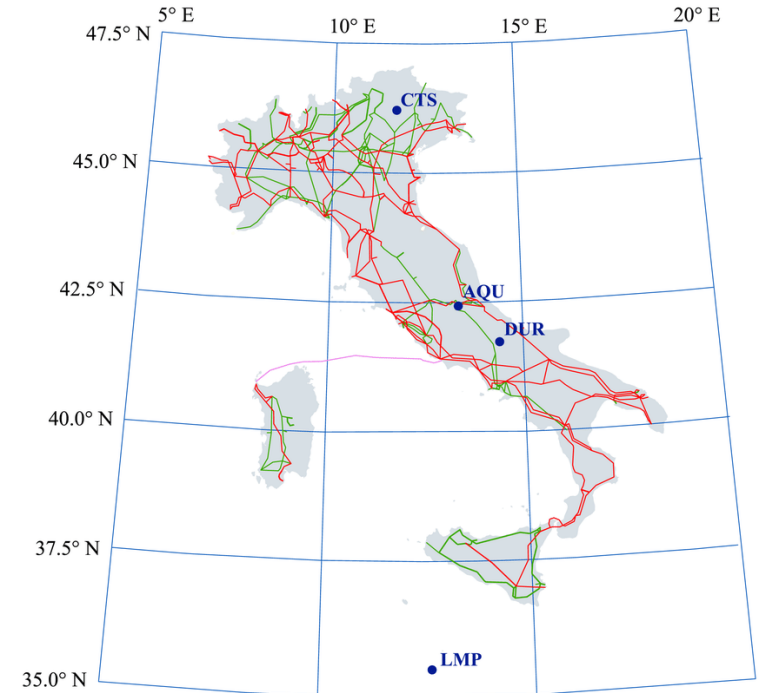
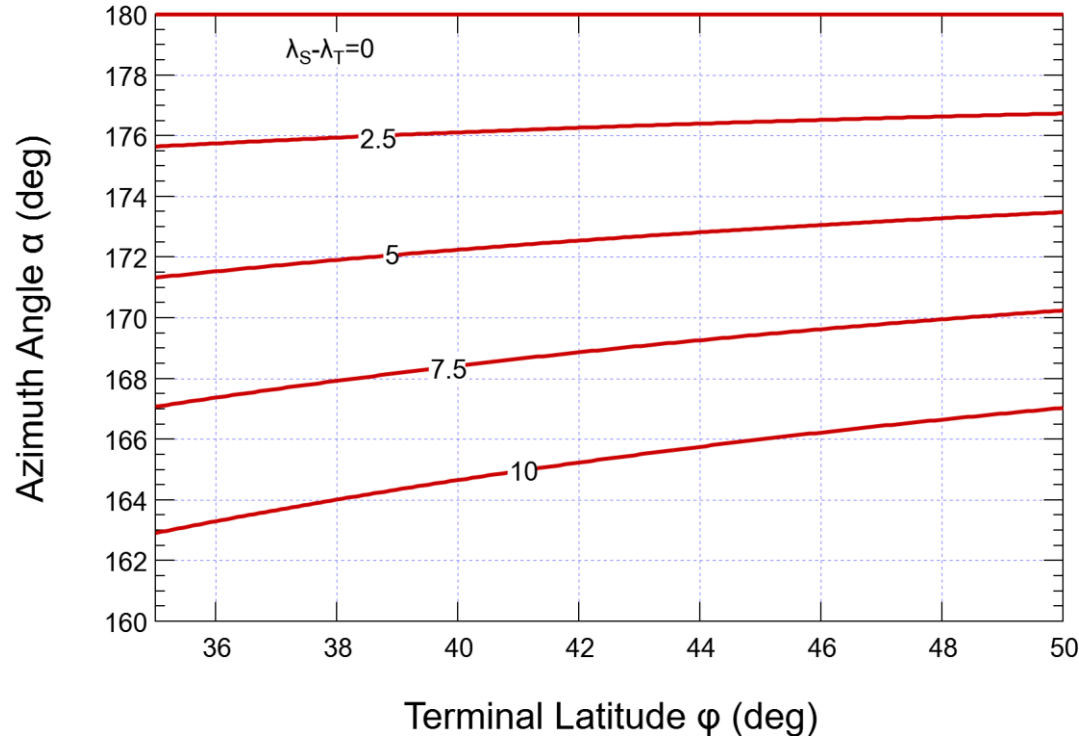


@DII for Hot Bird $\lambda_S - \lambda_T = 13 - 10.38 = 2.62$ deg

@DII, $\phi = 43.72$ deg



Azimuth Chart for Italy, looking EAST



@DII for Hot Bird $\lambda_S - \lambda_T = 13 - 10.38 = 2.62$ deg
 @DII, $\phi = 43.72$ deg, $\alpha = 180 - \alpha_0$ deg

How do they
launch it?

